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Exploring the Potential of Quantum Computing for Electrical Power System Optimization

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Abstract

In order to achieve the goals of the European Green Plan on zero net greenhouse gas emissions by 2050 and achieve a resource-efficient and competitive economy, significant changes are expected in the electricity market and power system management in Europe. By increasing the share of variable energy sources and increasing the number of participants with different behavioral patterns, transmission system operators are faced with the challenge of ensuring security of supply to end customers. Given that building a network is an expensive and time-consuming process, optimizing the existing network maximizes the potential of the existing network while increasing security. The optimization process is an iterative, computationally demanding process based on linear algebra, i.e., multiplication of large sparse matrices and mixed integer linear optimization, which in the operational environment is time-limited to a few minutes. It is precisely the optimization execution time with high costs of the server system that is the main obstacle and challenge for further integration of optimization into operational processes.
Recent advancements in quantum computing indicate its potential for significantly faster execution times than conventional optimization algorithms, particularly due to its ability to explore multiple solutions simultaneously (which is the main issue when grid optimization is performed using topological measures due to their physical nature preventing superposition) and efficiently handle complex mathematical processes inherent in optimization problems. However, realizing these benefits requires addressing challenges such as developing tailored quantum algorithms, implementing error mitigation techniques, and achieving scalability for real-world applications.
This paper presents basic concepts of quantum computing and state-of-the-art quantum computers available, as well as mathematical concept of so-called sensitivity analysis, congestion management and optimization of electrical power system. Overview of existing quantum algorithms designed for the purpose of linear algebra is given, with special focus on Harrow-Hassidimi-Lloyd quantum algorithm. Finally, paper provides assessment of applicability of existing quantum algorithms and quantum computing (in terms of available qubits) on the problem of real-world electrical power system optimization for which authors developed a solution based on conventional algorithms and closed mathematical optimization.

Introduction

The global mission to significantly reduce carbon emissions has become one of the main change drivers within the energy sector and has put in place extremely challenging tasks in front of energy sector actors. In order to reach the European Green Deal targets of no net emissions of greenhouse gases by 2050, as well as to achieve a resource-efficient and competitive economy, significant changes can be expected within electricity market behavioral patterns and consequently in the premise of how the European electrical power system is operated.

Based on the legal requirements and incentives in place, one can expect following changes in terms of electrical power systems operation: a) significantly higher percentage of the intermittent (renewable) energy sources within the energy mix; b) significantly increased number of electricity market participants with different behavior patterns (electric vehicles, smaller production units, demand side response, prosumers, aggregation, multi energy networks), even on the lower voltage levels; c) significantly increased amount of data to be exchanged in a short time and on different levels; d) higher percentage amount of the transmission lines' capacity provided at disposal to the market in order to facilitate the overall energy trading as well as to bring the objective closer to the real time, thus allowing a more efficient grid operation and utilization.

All these requirements and operational uncertainties require stronger grid infrastructure (e.g. investing in smart physical assets, electrical grid expansion, installation of flexible assets that possess high balancing potential) and/or better utilization of the existing one. Given that building a network is an expensive and time-consuming process, optimizing the existing network presents an efficient solution which maximizes the potential of the existing infrastructure while increasing security. The optimization process involves a series of iterative, computation-intensive steps grounded in linear algebra, specifically involving the multiplication of large sparse matrices in order to compute factors needed for mathematical optimization. To be more specific, so-called remedial action optimization (RAO) problem is a problem of the complexity class non-deterministic polynomial time (NP-Hard) and no efficient algorithms exist to find the global optimum guaranteed in polynomial time.

Recent developments in quantum computing suggest it could greatly accelerate execution times compared to traditional optimization algorithms. This is largely because quantum computing can process multiple solutions at once - a major advantage when optimizing grids using topological measures, where the physical nature of the measure's limits superposition - and adeptly manage the complex mathematical tasks that optimization problems typically involve.

Many organizations are heavily investing in quantum computing bearing in mind the potential disruptiveness it can bring to the market. According to Precedence Research [1], the global quantum computing market size is projected to hit around USD 125 billion by 2030 and is poised to reach a CAGR of 36.89% from 2022 to 2030.

Chapter 1 of this paper will present in detail the business process and explain the problem of grid optimization. Additionally, it will explain principles behind quantum algorithms and what are competitive advantages of it compared to classical algorithms.

Chapter 2 discusses various quantum algorithms for linear algebra.

Chapter 3 provides main conclusions and predictions regarding future of quantum computing application on the problem of electrical power system optimization.

1. Requirements for Grid Services - Technical & Market Conditions

In order to ensure security of the energy supply, transmission system operators today rely much on the accuracy of predictions of the market positions, and at the same time cope with uncertainties mainly by contracting different production units that can offer flexibility when deviation is present. These flexibility options are ordered by the system operator either because of the market or due to an unplanned incident within the grid.
Aiming to allow optimal electrical power grid utilization on the European level in terms of available capacity and operation cost, optimization of electrical power grid was introduced, and was designed in order to meet the requirements related to coordinated use of remedial actions (RAs) among system operators in a cost-efficient way, all documented under existing EU legislation: a) Commission Regulation (EU) 2017/1485 [2] – The Guidelines on System Operation; b) Commission Regulation (EU) 2015/1222 [3] – The Guideline on Capacity Allocation and Congestion Management; c) Commission Regulation (EU) 2019/943 [4] – Regulation on the internal market for electricity. Main use of the RAO process is foreseen in scope of Coordinated security assessment (CSA/ROSC) and Coordinated capacity calculation (CCC) processes, which both present the main building blocks of the European power system design for the new era.
The existing process' designs foresee extremely challenging time limits (see Figure below for details) for the execution of the electrical power grid optimization process, which are expected to be shortened in the future due to increasing requirements linked to the design of European Intraday energy market. The mentioned time limits seem to be unreachable with the current setup, even for the limited computational problem, while real time optimization of electrical power grid, which can be considered as a holy grail of system operators and would allow redesign and implementation of new concepts in the European energy market, is not foreseen at all at this moment due to problem size and complexity.
Overview of the current European energy market design is presented on the Figure below:

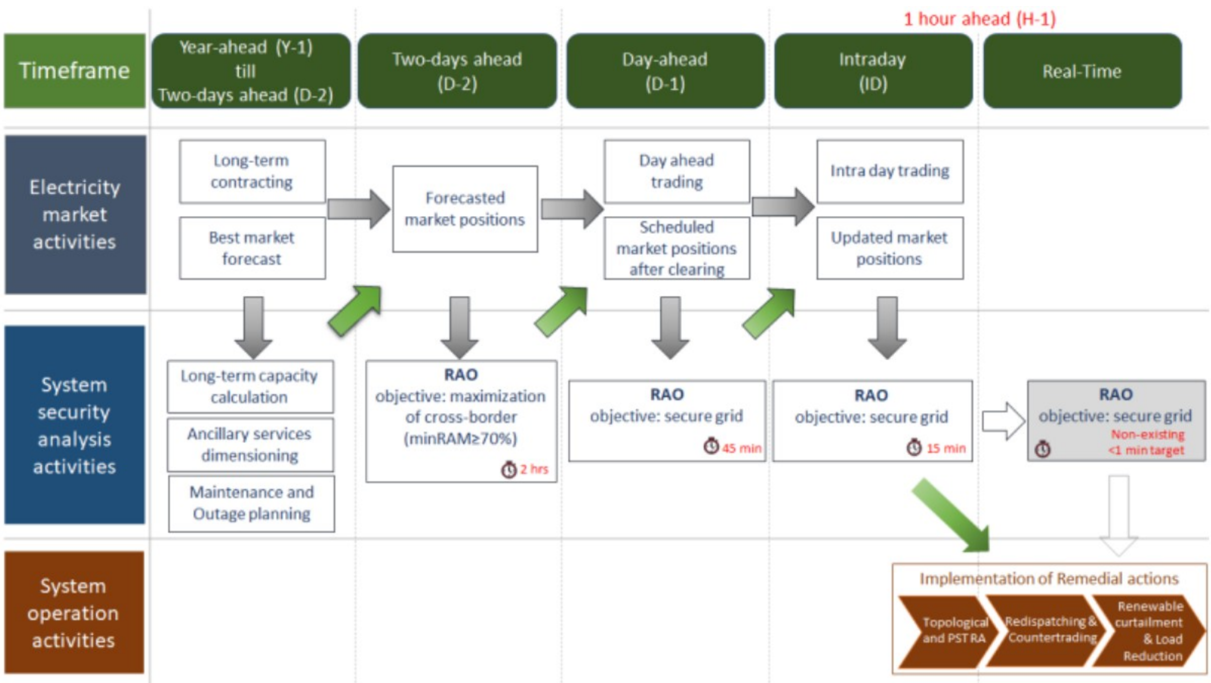

Figure 1 Overview of processes around European electrical power system operation

It is important to mention that once system operation reaches real time, the electrical power system operator is not focused anymore on the global optimum and cost of the

operation, but rather does his best to keep the system stable and ensure energy supply to all end users. In general, in case of security constraints identified in power system operation, three types of RAs are available to the power system operator: a) grid reconfiguration (topological RAs and PST tap changes) - non-costly; b) redispatching and countertrading – costly; c) Cross-border capacity and/or renewable generation curtailments and consumption reduction - last resort costly measure.
Additionally, one can easily conclude that with increased number of intermittent energy sources and number of market participants, forecasting of the market will become more complex and inaccurate, leading to: a) increased flexibility requirements – costly; b) increased number of redispatching and countertrading activations in real time - costly and not globally optimal; c) generation and load reductions – undesirable.
In the end, the following is going to reflect on the electricity prices for the end users, and likely to the unwanted usage of CO_2 emission units.

Considering that, depending on the process, minimal or no approximations can be applied to the optimization process, it is in principle divided into two main parts:

1. **Security analysis**: Main instrument to perform capacity calculation and analysis of the electrical power system stability in line with the forecasted and/or changed market positions, adjusted grid configuration due to unplanned unavailability of grid elements, etc. Within this step, load flow and sensitivity computations are performed for each outage, available remedial action (RA) and their combinations. The set of potential remedial actions, which system operators can apply to ensure stable grid operation, consist of the following: change of a control tap of power shifting transformer (PST), change of a topological status of a power grid element (e.g., switching on/off a high-voltage line, circuit braker or busbar coupler), change of production and/or consumption at a certain node in the grid (e.g., redispatching). Sensitivities describe how much load flow/voltage will change on a certain element if some other condition is changed - nodal power injection, tap of the PST, element status. Calculation of these factors requires significant time that can easily increase with new outages/elements being observed and number of RAs to analyze.
2. **Combinatorial optimization**: combinatorial optimization of the RAs and their effects as calculated in step one. In general, optimization shall find optimal set of RAs activations in order to obtain a state in which the maximum value of element load flows is as small as possible, as well as associated system operation costs. To reach it, optimization algorithms favor usage of non-costly RAs (topological RAs and PST tap changes), ahead of costly RAs (changing of power plants production - redispatching). But the decision is not only based on the factors calculated in step 1, but also multiple external business constraints imposed by TSOs must be considered.

As discussed earlier, the calculation of factors integral to optimization involves iterative solving of nonlinear (e.g. Newton-Raphson method) or linear equations (e.g. DC load flow) using linear algebra techniques. Each modification in the system topology necessitates recomputation of the system. Notably, in an operational environment, these topology changes can amount to as many as 80,000 modifications leading to inadequately high computation times (it would take around 4 hours to solve this problem with present software and hardware used in European capacity calculation process, while for ROSC time and implementation feasibility assessment still does not exist for the full problem). To tackle it, current computation hardware is robust and complex, especially considering high security requirements.
The nature of quantum physics and consequently quantum computing represents a compelling solution for this problem. A primary benefit arises from the inherent nature of

quantum states, which allows that particles are in superposition of multiple states simultaneously. For example, an electron in a quantum system can simultaneously occupy multiple positions or energy states until it is measured. Consequently, opposite to classical computing where information is stored in bits which can be either in state 0 or 1, quantum computing stores information in quantum bits, or qubits, which can be in superposition of 0 and 1. Mathematically, quantum state is described as vector in Hilbert space. Using Dirac bra-ket notation, which is common when representing quantum algorithm, qubit can be defined as:

$$a|0\rangle + b|1\rangle,$$

where probability amplitudes a and b are complex numbers, with normalization:

$$|a|^2 + |b|^2 = 1.$$

In addition to the superposition principle, another important quantum physics property is the quantum entanglement. It enables to reach so called quantum supremacy for some of the algorithms already with 70 entangled qubits [5], meaning that quantum computer with such number of qubits will solve some problems that classical computer cannot solve. In this entangled state, measuring one qubit immediately affects the state of the other. If we measure the first qubit and find it in the state $|0\rangle$, the state of the second qubit is also determined by this measurement (for example, $|0\rangle$). If the first qubit is in the state $|1\rangle$ upon measurement, the second qubit will also be found in the specific state (for example, $|1\rangle$). This correlation holds no matter how far apart the qubits are, a phenomenon that Einstein famously referred to as "spooky action at a distance". The number of possible quantum states for 2 entangled qubits is $4 = 2^2$. It is straightforward to see that the number of possible quantum states for n entangled qubits is 2^n. This exponential growth, combined with the possibility of superposition, provides the superiority of quantum computers. When entangling quantum states, the mathematical tool we use is the tensor product, which underlies exponential growth. In classical systems, this is contrasted with the Cartesian product, where growth is linear.

This is evident from the quantum algorithm proposed by Shor [6,7] for factorization which is exponentially faster than any known classical algorithm, and Grover's quantum search algorithm [8] capable of searching a large database in time square root of its size.

The efficacy of quantum parallelism is notable; however, a significant limitation arises during the measurement of outputs. Upon measuring a quantum system, its quantum wave function collapses, transitioning the system from a state of superposition to a specific, determinate state. To resume computations, the system must be reinitialized to the appropriate initial state. Moreover, the management of measurements presents further challenges. Qubit quality, coherence times and error rates pose critical hurdles. In order to tackle such issues, multiple entities (including Google, IBM and various startups) employ various error correction algorithms, along with the development of quantum algorithms as such. Additionally, the challenges of scaling up the number of qubits are so great that some researchers think it will require totally different hardware from the microelectronics used by the likes of IBM and Google, where coherence time is typically around a millisecond at best, meaning that all quantum computation must happen within that time frame.

Most notable qubit types are: a) Superconducting qubits [9] - Operating in low-temperature environments, this is currently the most advanced technology; b) Trapped ions [10] - A technique that manipulates individual ions to realize qubits. Trapped ion qubits are noteworthy for long coherence times as well as high-fidelity measurements; c) Topological

quantum computing [11] - A research-stage technology known for stronger error resistance; d) Neutral atoms [12] – Using lasers, these atoms can be excited into a number of quantum states, any two of which can be used to create a qubit that is well suited for scaling up and performing operations (e.g., Aquila quantum computer); e) Photons [13] – By setting and measuring the directional spin states of individual light particles, photon qubits can be used to send quantum information across long distances through optical fiber cables and are currently being used in quantum communication and quantum cryptography.
In terms of the state-of-the-art quantum computer, at the end of last year, IBM announced a chip with 1121 qubits called the Condor [14] and the company Atom [15] released a chip with 1125 qubits, but error correction remains the main challenge.

2. Approach towards Flexibility and Business Experiences and success stories

As discussed in the previous two chapters, considering the intermittent nature of renewable energy sources, grid flexibility and better utilization of it will become increasingly important to maintain the security of supply in the future. Quantum computing as enabling technology could be one of the answers on how to efficiently optimize power system operation, even close to real time.
Having a tool that can provide optimal grid configuration and generation outputs should reduce flexibility requirements, and lead to more efficient wholesale markets and less curtailment needs.
Since current system optimization is based on the (iterative) solving of linear equations using DC load flow method, one of the quantum algorithms that could be used for this problem is Harrow-Hassidimi-Lloyd (HHL) quantum algorithm which theoretically provides exponential speedup [16]. HHL algorithm can be used to solve equation

$$A|x\rangle = |b\rangle,$$

where $A \in \mathbb{R}^{N\,x\,N}$ is Hermitian matrix that is equal to admittance matrix vector $|x\rangle \in R^{Nx1}$ is the desired solution, i.e. the quantum state which encodes the power system nodal angles in its amplitudes, and vector $|b\rangle \in R^{Nx1}$ is the quantum state which encodes the known power system nodal balances in its amplitudes. The HHL algorithm run-time is theoretically equal to:

$$O\left(\frac{s^2\kappa^2}{\varepsilon}\log(N_b)\right),$$

where N_b is number of linear equations, κ is (Hermitian matrix) system condition number, s is matrix sparsity and ε is accuracy of the solution.
The HHL algorithm has been experimentally implemented on various physical platforms, and it can now be performed on publicly accessible quantum computing platforms, such as the IBM Quantum Experience [17] using Qiskit software development kit, whose qubits are made from so-called superconducting transmons.
Main issues identified related to application of HHL algorithm for security analysis problem include: a) Need for quantum memory to perform iterative HHL-based quantum algorithms and achieve computational speedup; b) Need for more shallow-depth quantum circuits to mitigate noise impact and scalability issues; c) Need for a matrix preconditioning and/or filtering functions to address computational issues caused by ill-conditioned power systems; d) The process of extracting the values of nodal angels from the quantum state $|x\rangle$ reduces the speedup provided by quantum computers [18], therefore there is a need to formulate the problem in such a way that the quantities of interest are scalar products and

expectation values, which would reduce the number of measurements needed to obtain useful information from the solution provided by the HHL algorithm.

There are also other novel quantum algorithms being developed that can solve linear equations, such as algorithms based on adiabatic quantum computing (AQC) [19] or (variational) hybrid quantum-classical algorithms (VHQCA) [20].

Briefly, in AQC we assume that the Hamiltonian of a certain quantum system depends on some parameter which changes in time, and that the system is initially in one of its non-degenerate eigenstates with an energy gap to other eigenstates. The adiabatic theorem ensures that such quantum system evolves in time in such a way that it remains the eigenstate of the current Hamiltonian, under the condition that the change of the Hamiltonian is sufficiently slow (adiabatic) and that there is always an energy gap. In other words, in the adiabatic process the eigenstate of the initial Hamiltonian continuously evolves into the eigenstate of the final Hamiltonian. In the context of quantum computing, we can imagine the final state of the system as a state which contains information about the requested the solution to the problem. In Ref. [19], the authors proposed two algorithms for solving the linear system, i.e., preparing the quantum state $|x\rangle$ which is proportional to the solution of $A|x\rangle = |b\rangle$. The advantage of such algorithms is that they do not use a phase estimation algorithm, nor an amplitude amplification algorithm, so a large number of ancillary (ancilla) qubits are not required.

VHQCA algorithms are interesting in the context of reduced number of qubits requirements since optimization of variational parameters is performed on a classical computer. As a result, the number of required quantum gates, or the depth of the quantum circuit, is reduced, which leads to a reduction in noise. For example, the VHQCA strategy has been applied in Ref. [20] where it has been demonstrated that even larger systems of linear equations (of dimensions 1024x1024) can be solved with such approach.

In addition to HHL, AQC, VHQCA and other algorithms that would be used for sensitivity analysis which requires solving linear systems of equations, the optimization procedures for electric power systems generally need methods for solving nonlinear systems and can be defined with generic objective function $f(x)$ that has to be minimized. In this context, two important quantum optimization algorithms are quantum gradient descent method and the quantum Newton's method [21].

The gradient descent method seeks the point of minimum by starting from an arbitrary initial value x^0 and moving the argument of the target function in the direction of the negative gradient of the target function. The process continues in each subsequent step, according to the expression:

$$x^{t+1} = x^t - \eta \nabla f(x^t),$$

where $\eta > 0$ is parameter that can be adjusted during the iterative procedure.

In Newton's method, the iterative process also takes into account the curvature of the target function, namely the Hessian matrix H, whose elements are $H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$, with a step:

$$x^{t+1} = x^t - \eta H^{-1} \nabla f(x^t),$$

where matrix element H_{ij} is calculated in point x^t.

The quantum version of these optimization algorithms described in [21] focus on sparse homogeneous polynomials, requiring the vector x to be normalized and utilizing qubit-based representations for efficiency. These algorithms, including quantum versions of gradient descent and Newton's methods, are designed to scale exponentially with the number of iterations yet poly-logarithmically with the vector dimension, suggesting they are effective for large systems where initial approximations are near true solutions. Challenges include their complexity and the need for additional quantum computational techniques like phase estimation and matrix operations, with their practical application and broader adoption dependent on future advances in quantum computing technology.

Note: We note that very recently several implementations of power-flow analysis in electric systems with quantum computing approaches have been reported (see Ref. [22] for the most recent one, and refs. therein). However, such calculations have been limited to small-scale test systems (for example, with up to 14-bus test system in [22]).

3. Added Values/Conclusion

This paper provides overview of the main challenges of power system operation where amount of the renewable energy sources in the energy mix is significant, where typical generation and load patterns do not exist anymore since more and more grid users act as prosumers and their number increases significantly. With the goal of better grid utilization and enhanced security, optimization of the grid operation has been introduced into multiple European grid and market operation processes. However, the optimization problem has been quite simplified due to the complexity of mathematical operations and consequently significant hardware requirements. Further advancements in this area are therefore slowed down or completely blocked.

Quantum algorithms and quantum computing become more and more advanced and stable, leading to the fact that they must be considered as enabling technology that could potentially bring breakthroughs to grid optimization.

This paper also discussed state-of-the-art quantum computers and quantum algorithms for linear algebra and linear optimization, with focus on HHL and Newton-Raphson quantum algorithms.

Main question to be answered though is how far is the point in time when quantum computing could be used for operational power system purposes. If the noise, i.e., errors in qubits, were small, computers with about 50 entangled qubits could compute the problem discussed using HHL algorithm in significantly reduced time [5]. However, since the noise (or random errors on qubits) is still significant, it is necessary to use what is known as the quantum error correction code, or schemes for error correction that require redundancy in the number of qubits needed for computation. Colloquially speaking, several entangled qubits are needed to record 1 qubit of information (redundancy), and a so-called quantum error correction scheme. Therefore, to perform the HHL algorithm for matrices of the order of size 100,000 squared, 50 entangled qubits need to be increased several times, and such computers do not exist today. As mentioned in the paper, just at the end of last year IBM released Condor with just over 1000 qubits, while their industry-defining roadmap for a decade of quantum innovation sets ambitious targets in terms of number of quantum gates and qubits developments until 2033.

Finally, Europe started to invest more in the quantum computing and IBM will complete a new EU quantum data center in Ehningen, Germany, in 2024, bringing local quantum resources to Europe [23].

The authors of this paper strongly believe that within the next 10 years there will be a quantum computer available able to solve the power grid optimization problem. Bearing in mind the importance of secure electricity supply and the complexity of grid optimization, there is a huge potential for application of quantum algorithms for RAO problem. Unfortunately, no one yet has been able to convincingly demonstrate a quantum advantage in solving large-scale power system problems mainly because of noisy intermediate-scale quantum (NISQ) era, where quantum devices are noisy and have limited quantum resources.

Therefore, the Authors strongly believe that development of scalable quantum algorithms which will be able to solve real size RAO problem must be put in focus now to be prepared once hardware is ready, especially considering disruptiveness it brings.

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